THE MATHEMATICS TEACHER

EDITED BY
W. H. METZLER

ASSOCIATED WITH

EUGENE R. SMITH HARRY D. GAYLORD
GEO. GAILEY CHAMBERS WILLIAM E. BRECKENRIDGE

VOLUME XIII DECEMBER, 1920 NUMBER 2

EDUCATIONAL OPPORTUNITY IN THE ARMY OF OCCUPATION.

BY J. T. RORER.

(*Concluded from page 37*)

or forty men whom he distributed among the divisions as organizers of schools, special teachers, lecturers, etc.

The educational work of the A. E. F. was firmly established by General Orders 9 and 30, issued by the Commander in Chief. Among other things in these orders, General Pershing authorized three types of educational assignment for the men: first, at universities or colleges; second, at divisional schools; third, at post schools. Only the work for illiterates at post schools was to be compulsory, but the elective opportunities attracted far more applications than could be granted. Thousands of men took advantage of the remarkable privilege to study at the French and English universities and professional schools, while on the army payroll, and free of any personal expense. These in the main were men who had been in American colleges and universities, or who were fully prepared for university work when they left home. Shortly after they began their work at Cambridge, Oxford, Edinburgh, Paris, Grenoble, or any of the other established seats of learning, another call was issued announcing the opening of the American University at Beaune. Cote d'Or. Here was an educational miracle; a Harvard and

Chicago combined, thirteen colleges completely organized almost over night and surprisingly well equipped, with a wonderful faculty of about one thousand, and a student body of more than eight thousand. During the three months that it was in session, it gave an intellectual impetus to thousands of young men and undoubtedly was a large factor subsequently in bringing many of these students back to home colleges and universities.

The Army of Occupation sent its full quota to the universities. Complete data are not at hand that the reader may have accurate statistics, nor has the government issued final reports. The Second Division sent over 300 men to Beaune and to the other universities. If each of the other occupational divisions did as well, there were between two and three thousand of these troops in attendance at the universities. Of course several thousands of additional students were detailed to the universities from the million or more American troops in France at that time.

While the importance of the university work cannot be overestimated, it must be remembered that but comparatively few men received its benefits when the immense size of our army is considered. A far larger group was reached by the post and divisional schools. General Orders No. 9 provided that instruction was to be offered in the following branches in the post schools: Common school subjects, modern languages, United States history, history of England and modern Europe, civics and citizenship, and in any other subject authorized by G. H. Q. Each commanding officer was instructed to appoint a qualified school officer, usually a chaplain, to organize the classes, appoint teachers, and supervise the instruction. Text books were to be supplied by the Y. M. C. A., and the Y educational men were to function as general superintendents and directors, in an expert advisory capacity.

In the organization of the work in the Army of Occupation, Dr. Benton, the regional director, closely followed the usual military arrangement. The three corps, the 3d, 4th, and 7th, each had its corps director; each corps director supervised the work of two or three divisional directors; each divisional director was aided by several assistants and specialists. Each directing educational man worked in close conjunction with an army school officer who issued the necessary orders to initiate

and carry on the work. The director, for instance, would select the teachers from the available candidates, or textbooks from standard lists; his co-working school officer would issue the orders that would move teacher, Corporal Jones to his class, and the books from a warehouse to the men who needed them. Thus in general, the organization became a duplex system which often seemed cumbersome and wasteful of time and effort. However, to one experienced in a large city school system, army red-tape was no more elaborate, generally speaking, than the modern public school variety.

The army became greatly interested in the work and usually there was friendly co-operation between army officials and the educational men. Army officers are human, and though naturally they expect a special etiquette,—courtesy, good humor, fidelity, and common sense appeal to them in the same way that these qualities do to civilians. There was a noticeable absence of the exaggerated self-importance so often attributed to the officer; fawning, bootlicking, and kowtowing were held in proper contempt. On the other hand, these officers, most of whom were "regulars," were indulgent of our ignorance of military life, and generally were desirous to work with us harmoniously for the good of the service.

The post school announcements soon aroused a great demand. Cafés, halls, and large rooms of dwellings were turned into school rooms. Sometimes schoolhouses were available; but these frequently had been pre-empted previously for headquarters of battalion, regiment, or division, and the schools could not dislodge the headquarters. At first supplies were slow in arriving, but later an abundance of paper, pencils, text and reference books on a great variety of subjects were at hand.

The removal of illiteracy from the army was a prime motive of the post schools. Commanding officers were instructed to order all illiterates to the classes at least two hours per day. Men who could not sign the payroll made much unnecessary trouble and extra work at headquarters; for this reason alone, many officers were as careful to require them to report in the schoolroom as on the rifle range. But most of the men needed no urging, only here and there some sensitive doughboy, often of foreign birth, was content without some knowledge of written English.

The work of the post schools was by no means confined to elementary English and arithmetic. Many a young lieutenant studied algebra, geometry, and trigonometry to fit himself for promotion. Geography and history, especially of Europe, were popular; and the languages of these countries received considerable attention. Post schools were easily established where there were one hundred or more soldiers; indeed, sometimes there were eager young teachers, generally among the "noncoms," who would start a class in a lonely post where were stationed as few as twenty men. If the sergeant in command was an enthusiast in some subject, he could soon recruit a class. A request to the school center by telephone or messenger would bring the textbooks, and probably a teacher's manual, reference books, and the necessary supplies.

When the post schools were well under way, General Order, No. 30, authorized the establishment of the divisional schools, which were the high schools and vocational schools of the system. Like the universities, these schools attracted many more list of applicants, the commanding officers would detail to the temporary school battalions such men as could be spared, not to exceed in any case fifteen per cent. of the command.

Our divisional school was established at Rengsdorf, a Westerwald health resort. The cafés, exchanges, and dining-rooms of the many large hotels and pensions afforded excellent classrooms, and the comfortably furnished bedrooms were the best billets the boys had found since leaving home. Two large messhalls were the goals of the "chow lines," after the men had received their generous allotment of "corn-willie" and other army dainties from the kitchen police, in the best cafeteria style—but there was no cashier to detain the line. In the messhall tables with benches provided unexpected comfort; while the teachers in recognition of their dignity were seated on chairs surrounding tables covered with white cloths! Officers and educational men lived in the best pensions and received similar treatment to that formerly accorded by those establishments to their guests.

The divisional school was a modern military school with a very extensive program including vocational subjects. The uniformed students and officers, the daily one hour drill, the

men than military necessities would permit to attend. From the bugle calls, the usual camp life and organization gave an unavoidable military atmosphere. But as far as the class-rooms and the school work were concerned, the spirit was that of a good up-to-date school of the old peace times. Teacher and students worked together on a friendly basis,—rank made no difference and salutes were forgotten.

The daily schedule of Private Jones will indicate the busy life of the divisional school students:

	A. M.
First call	5:45
Reveille	6:
Calisthenics	6: – 6:15
Mess	7:
(Sick call)	7:30
First period class (French)	8: – 8:50
Second period (Europ. Hist.)	9: – 9:50
Third period (Economics)	10: –10:50
Assembly	10:55
Drill	11: –11:55
Mess	12: m
Fourth Period (English)	1:15– 2:05
Fifth Period (Salesmanship)	2:15– 3:05
Athletics, etc.	3:15– 5:15
Mess	5:30
Call to quarters	9:45
Tattoo	9:55
Taps	10:

There were no classes on Saturdays and Sundays. Frequent lectures, concerts, regimental shows, and movies were given in the evening. It will be noticed that there were five school periods of 50 minutes each, with ten minutes necessary intermission between successive periods to enable the student to meet consecutive appointments in remote buildings. The standard program contained four or five studies arranged in consultation with one of the educational men. These subjects were offered: bookkeeping and accounting, arithmetic, English, commercial law, commercial geography, salesmanship, stenography, algebra, geometry, trigonometry, calculus, civics and economics, general science, psychology, history, literature, French, Spanish, Italian, German, and drawing.

The location of our agricultural school at the same center made it possible for both groups of students to elect from the two programs under proper limitations. So these additional studies were available: farm management, soils and fertilizers, animal husbandry, cotton raising, dairying, forestry, horticulture, poultry raising, and market-gardening.

Our teachers were selected from the division at large without regard for rank, but generally they were non-commissioned and the lower ranks of commissioned officers. Mostly they were college men; many of them had taught at home; all possessed keen minds, good common sense and enthusiasm for the work. Under the supervision of the trained educational men, they rapidly became a strong group of teachers.

School libraries of over 1,000 titles were made possible by the assistant of the American Library Association which deserves generous praise for the quality and quantity of its service. The large library, reading room and study hall in what was formerly the dining-room of the "Zum Stern," became one of the conspicuous features of our Rengsdorf schools. Had the occupational army remained on the Rhine several years, our libraries would have provided ample intellectual rations in almost all fields of thought.

The wide interests of the men made numerous activities possible. Baseball, social and special clubs, hikes, Rhine trips, movies and other shows, all were popular. Even the followers of Isaac Walton had new stories to tell about speckled trout recently taken from those sparkling streams!

Most of the vocational courses were given by the several units of the division most closely related to the work. Thus the engineers gave the courses in road construction, carpentry, surveying, sign painting, photography, etc. The signal corps gave courses in telegraphy, wireless, telephony, and practical wiring. In other units we had courses in baking, cooking, barbering, horse shoeing, and motor mechanics.

When the popularity and success of the educational work was assured, the educational department of the Y was transferred to the army under the caption "Educational Corps." Most of us could feel but little change. The red triangles were ripped from our clothing and the Sam Brown belt was added.

We were instructed to draw our pay from the Quartermaster, but the Quartermaster would not receive us as any of his,—result, no funds until official announcement of our army existence permeated from G. H. Q. to our important Q. M.'s. Our changed status made no appreciable difference in our work; those who scorned us before as Y men, now despised us as field clerks. These scorners in the main were a few "shave-tails" whom G. H. Q. neglected, somehow, to consult about the management of the army! Like ourselves, however, the officers generaly recognized that the Educational Corps was a temporary organization designed to do a definite piece of work that was well worth the doing, and they continued the excellent support they had given us as Y men.

The few months of this strange life in Germany now seem like a dream. Two hundred thousand young Americans in what was once one of the gardenspots of the kaisers domain! Here they were living in comfort, some in the very castles that were the pride of his autocracy, some hunting and fishing in his majestic forests, some joy-riding over his well-made roads, some exploring the Rhine in his well-made side-wheelers, and some setting up American schools in his villages and towns! Did not the 17th Field Artillery establish a complete motor school at Ehrenbreitstein, the Gibraltar of the Rhine, where more than a thousand expert mechanics were trained? Did not the 2d Engineers establish a wonderful technical school at Engers-am-Rhein where they made use of the equipment and building (an old schloss) formerly used by one of the officers' training academies of the German army? Did not the American flag fly over these and many other similar citadels? When such new things under the sun were taking place, can you say that Germany did not know that she was defeated?

In May, 1919, the army was embarking for home with increasing rapidity. Division after division had left the Rhine. About this time it was decided to clear France of Americans. The Beaune University announced its closing. Unexpectedly the homeward order was extended to all the Educational Corps including those on the Rhine. But our schools were so well organied and the work had become so popular with the men that the school officers and teachers continued the classes until

the order came in June for the division to move to the very edge of the neutral zone and to be in immediate readiness to march toward Berlin. This of course broke up the schools. After the treaty was signed two weeks later, the division entrained for Brest. Soon the transports conveyed the several units to New York, where they marched in review on August 9th.

And was it all worth while? To evaluate the results of the educational campaign is not easy; the time was too short to appraise it in terms of our common scholastic units; and yet it would be difficult to find results that were more genuine and far-reaching. The men were very outspoken in their appreciation; they studied earnestly and their accomplishments especially in vocational subjects were marvellous. The long absence from books seemed to heighten the interest of those intellectually inclined. For the first few days they were dazed by the printed page, so rarely seen during the strenuous times of the war. But the intellectual starvation through which they had passed created a hunger that could be but partially satisfied in our army schools, and that was bound to express itself in continued study and training when they reached home. The thousands of service men at present working in our colleges, universities, professional and technical schools is one endorsement at least of the work of the educational corps.

Another endorsement is the effect on the army and navy itself. Nearly every recruiting poster emphasizes education: JOIN THE ARMY TO SECURE AN EDUCATION FREE; ENLIST IN THE ARMY AND LEARN A GOOD TRADE; and one sees similar catchy sentences for the Navy.

Education is a part of army camp life to a far greater degree than ever before. It is now the aim to return every soldier to civil life not only in better physical condition than when he enlisted, but also better equipped to meet intelligently the duties of an educated citizen.

These results seem to me to warrant fully the large expenditure of the Y, the A. L. A., and the government on education in the A. E. F.

William Penn High School,
Philadelphia, April 24, 1920.

THE OUTLINE METHOD IN MATHEMATICS.

By Robert R. Goff.

Printed outlines have been used in many school subjects for some time, and, I think, have in general been found helpful. Moreover, by excluding the non-essentials of a subject, these outlines can hold up for closer inspection and emphasis the essentials of that subject. This is surely a strong point, and one that the large textbooks cannot claim. Work of this sort has been carried on in English, history, geography, civics, etc., but the secondary mathematical field has been comparatively neglected. I have for some years been using, in manuscript or printed form, outlines in algebra and geometry. These outlines are made the basis of the recitation and the assignment. The original textbook then becomes a reference and supply book.

Sample pages of algebra and geometry follow:

FACTORING.

Factoring is the reverse of Special Products. There are four common cases.

1. Any number of terms from which a common factor can be divided.
Formula: See case 1, special products.

2. Two terms in the form of the difference of two squares.
Formula: See case 2, special products.

3. Three terms in the form of the quadratic trinomial.
Formula: See case 5, specal products.

4. Two terms in the form of the sum or difference of two like odd powers.
Formulas:

These four cases cover all the common methods of factoring. In any case the terms can be simple or parenthetical.

Case 2 can have three, four, or six terms, but they must then be grouped into two terms.

Case 3 can have four, five, or six terms, but they must then be grouped into three terms.

In case 4, even exponents can be considered odd except powers of 2. Thus:

ANOTHER METHOD OF FACTORING IS THE FACTOR THEOREM: If a polynomial in X equals zero when A is substituted for X, then X—A is a factor of the polynomial.

THE FOUR STEPS IN SYSTEMATIC FACTORING ARE:

1. Try case 1.
2. If there are only two terms, try case 2, or 4.
3. If there are only three terms, try case 3.
4. Try grouping into these cases, or else the factor theorem.

QUESTIONS ON FACTORING.

1. Algebraic factoring is the reverse of what?
2. State the four common cases of factoring.
3. State the formula of each.
4. Explain how polynomials of more than two terms can sometimes be factored by the second case.
5. Explain how polynomials of more than three terms can sometimes be factored by the third case.
6. What even exponents can be considered odd, if desired?
7. State the factor theorem.
8. State the four steps in systematic factoring.
9. If both case two and case four can be used, which is to be preferred?

CHAPTER XIX.

PROPORTION FROM SIMILAR TRIANGLES.

THE CORRESPONDING SIDES OF SIMILAR TRIANGLES ARE IN PROPORTION. (259)

269. *The corresponding altitudes of two similar triangles are in proportion with any two corresponding sides.*

271. *If two chords intersect within a circle, the product of the parts of one chord equals the product of the parts of the other chord.*

272. *If two secants intersect without a circle, the product of one whole secant and its external part equals the product of the other whole secant and its external part.*

273. *If a secant and a tangent intersect without a circle, the product of the whole secant and its external part equals the square of the tangent.*

276. *If a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse:*

   1. *The triangles formed are similar to the given triangle and to each other.*
   2. *The perpendicular is the mean proportional between the parts of the hypotenuse.*
   3. *Either arm is the mean proportional between the whole hypotenuse and its adjacent part.*

277. *The perpendicular from any point in the circumference of a circle to the diameter is a mean proportional between the parts of the diameter.*

PROBLEM: Construct a mean proportional between two given straight lines.

SUMMARY.

279. State two new methods of proving lines in proportion. Recall the other two methods.

280. State three methods of proving a line a mean proportional between two other straight lines.

DISCUSSION (OF CHAP. XIX).

The proof of lines in proportion from similar triangle has two parts: (1) Proving triangles similar by one of the summary, Art. 267; (2) The corresponding sides of similar triangles are in proportion.

The proof that the product of two lines equals the product of two other lines has three parts: (1) Proving triangles similar; (2) Corresponding sides in proportion; (3) Product of the means equals product of the extremes, Art. 241.

269. Method is 259. The same thing can be proved of medians.

271. Method is 259 and 241. Join the ends of two chords.

272. Method is 259 and 241. Join the alternate intersections.

273. Method is 259 and 241. Join the intersection points.

274. The last three theorems can be grouped under one statement. If two chords intersect (internally or externally), the product of the parts of one chord equals the product of the parts of the other.

276. Method is: (1) Art. 265; (2) Art. 259; (3) Art. 259.

277. Method is 276, 2.

278. Method is 277.

279. The four methods are 244, 249, 259, and 274.

280. The three methods are 273; 276, 2; 276, 3.

The teacher introduces a topic by discussing its nature and methods as in the outline. Comparison, contrast, etc., should be used in relating the new to the old. The pupil then makes a model under the teacher's guidance and inserts it in the outline. This is followed by exercises of this type.

In algebra there is usually room on the pages for the models. In geometry any proof, if desired, is written on plain paper and inserted. There are eight theorems in geometry whose proofs are not always interchangeable in the common textbooks. Each of these has references to books where a proof suited to the order of the outlines can be found.

Results from the use of these outlines seem to show that the pupil is usually interested in making his notebook. It is his own

handiwork, and he will often be found working on this when he should be doing something else. Second, if he has difficulty at any time with his exercises, he knows where to look for his model, which he has made and which he is, therefore more likely to understand. Third, he gets a clear view of a topic as a whole, and of its methods. He cannot emphasize non-essentials, because they are not in sight. Fourth, there is sound training in habits of systematic effort. He is taught to classify the work he is about to do, and then recall the methods of that type.

ACADEMIC HIGH SCHOOL,
NEW BRITAIN, CONN.

GREEK PHILOSOPHERS ON THE DISCIPLINARY VALUE OF MATHEMATICS.

By Florian Cajori.

Two independent methods have been used for the determination of the disciplinary value of mathematics: (1) the method of laboratory measurement, which, though ambitiously pursued, has not yet yielded altogether consistent results;[1] (2) the historical method which recounts the experiences and opinions of the great teachers and philosophers of past ages. It is the latter method that we wish to consider in this paper, particularly in reference to Greek testimony.

Certain writers of recent time have thrown dust into the eyes of teachers by endeavoring to set forth as a fact that Plato and other great thinkers of Greece denied the disciplinary value of mathematics in the general education of youth. These distortions have resulted from a paucity of quotation, combined with a failure to explain the technical uses of certain words quoted.

In his "Republic," Plato explained the education which citizens of his ideal State should receive. When he speaks of "arithmetic" and "geometry," he uses terms which had the same meaning then as they have now. But "mathematics" and "reason" conveyed different meanings then—the former receiving a broader, the latter a much narrower interpretation. With Plato "mathematics" included not only arithmetic and geometry, but also astronomy and probably other physical sciences; "Reason" was used in a very specialized sense. He divides intellectual knowledge into two subgroups ("Republic," VI., 510B, 511): "hypothetical knowledge" and knowledge of

[1] This is a special consideration under the general problem of the transfer of training. See H. O. Rugg, *Experimental Determination of Mental Discipline in School Studies*, Baltimore, 1916. J. W. A. Young, "Concerning Experiments to test the transfer of training" in *School Science and Mathematics*, Vol. 18, 1918, pp. 1–10.

"principles." He postulated the existence of two corresponding "faculties in the soul"—"understanding" and "reason." "Understanding" grapples with "hypothetical knowledge," "reason" with knowledge of "principles." To "reason" was to engage in dialectic (see Century Dictionary) *i.e.*, to draw *probable* conclusions, to determine the value of opinions, as opposed to the process of drawing *necessary* conclusions. The content of dialectic was largely ethics—"the good." Plato properly called mathematics "hypothetical knowledge." He taught that in the study of mathematics we exercise the "understanding," not the "reason"; we draw conclusions that are, as a rule, *necessary*, not *probable*. Plato ("Republic," VII., 531C) represents Socrates and Glaucon as engaged in the following dialogue:

"Now, when all these studies [arithmetic, geometry, astronomy] reach the point of intercommunion and connection with one another, and come to be considered in their mutual affinities, then, I think, but not till then, will the pursuit of them have a value for our objects [dialectic]; otherwise there is no profit in them. I suspect so; but you are speaking, Socrates, of a vast work. What do you mean? I said; the prelude of what? Do you not know that all this is but the prelude to the actual strain which we have to learn? For you surely would not regard the skilled mathematician as a dialectician? Assuredly not, he said; *I have hardly ever known a mathematician who was capable of reasoning.*"

What does Plato mean by this passage? Simply this, that a mere mathematician is not skilful in drawing *probable* conclusions, in the formation of opinions in metaphysical argumentation. He does not say that mathematical study is a hindrance to dialectic; quite the contrary; it is the opening door to dialectic. Mathematics is the "prelude," it is "but the prelude to the actual strain which we have to learn."

We proceed to give quotations from the dialogue in Plato's "Republic" which leave no doubt as to the rôle that arithmetic and geometry play in the training of citizens in his ideal State. In "Republic," VII., 522B, he says:

"We shall have to take something which is not special, but

of universal application. What may that be? A something which all arts and sciences and intelligences use in common, and which every one first has to learn among the elements of education. What is that? The little matter of distinguishing one, two, and three—in a word, number and calculation:—do not all arts and sciences necessarily partake of them? Yes. Then the art of war partakes of them? To be sure. . . . Can we deny that a warrior should have a knowledge of arithmetic? Certainly he should, if he is to have the smallest understanding of military tactics, or indeed, I should rather say, if he is to be a man at all. . . . (VII., 525B): The man of war must learn the art of number or he will not know how to array his troops, and the philosopher also, because he has to rise out of the sea of change and lay hold of true being, and therefore he must be an arithmetician. . . . Then this is a kind of knowledge which legislation may fitly prescribe; and we must endeavour to persuade those who are to be the principal men of our State to go and learn arithmetic, not as amateurs, but they must carry on the study until they see the nature of numbers with the mind only; nor again, like merchants or retail-traders, with a view to buying or selling, but for the sake of their military use, and of the soul herself. . . . I must add how charming the science is! and in how many ways it conduces to our desired end, if pursued in the spirit of a philosopher, and not of a shopkeeper! . . . (VII., 526B): And have you further observed, that *those who have a natural talent for calculation are generally quick at every other kind of knowledge; and even the dull, if they have had an arithmetical training, although they may derive no other advantage from it, always become much quicker than they would otherwise have been. . . .*"

"We are concerned with that part of geometry which relates to war; for in pitching a camp, or taking up a position, or closing or extending the lines of an army, or any other military manoeuvre, whether in actual battle or on a march, it will make all the difference whether a general is or is not a geometrician. Yes, I said, but for that purpose a very little of either geometry or calculation will be enough; the question relates rather to the greater and more advanced part of geometry—

whether that tends in any degree to make more easy the vision of the idea of good; and thither, as I was saying, all things tend which compel the soul to turn her gaze towards that place, where is the full perfection of being, which she ought, by all means, to behold. . . . (VII., 527C): Nothing should be more sternly laid down than that inhabitants of your fair city should by all means learn geometry. Moreover the science has indirect effects which are not small. . . . In all departments of knowledge, as experience proves, any one who has studied geometry is infinitely quicker of apprehension than one who has not. . . ."

That mathematical subjects are a necessary preliminary to the higher kinds of knowledge is brought out still more sharply in the folllowing passage ("Republic," VII., 536D):

"Calculation and geometry and all the other elements of instruction, which are a preparation for dialectic, should be presented to the mind in chidhood."

In another book, his "Laws" (V., 747), Plato sets forth the value of arithmetic in producing mental keenness, thus:

"The legislator is . . . to bid the citizens, as far as possible, not to lose sight of numerical order; for no single instrument of youthful education has such mighty power, both as regards domestic economy and politics, and in the arts, as the study of arithmetic. Above all, arithmetic stirs him up who is by nature sleepy and dull, and makes him quick to learn. . . ."

To sum up, then, Plato favored the study of arithmetic and geometry: (1) as not only necessary in war, but of universal application—hence to be prescribed by legislation; (2) as having great mind-training value, not for the pursuit of more advanced mathematics alone, but for the highest kind of knowledge, for dialectics—persons talented in calculation and geometry being "generally quick at every other kind of knowledge and even the dull . . . always become much quicker than they would otherwise have been"; (3) for the sake of "the soul herself."

We are informed by Tzetzes, a twelfth-century grammarian and poet of Constantinople, that Plato had an inscription over the entrance to his Academy at Athens: "Let no one who is unacquainted with geometry enter here." The authenticity of

this statement has been attacked on the ground that neither Plato nor his contemporaries and immediate successors mention this inscription. Diogenes Laertius (about 200 A. D.) tells a similar story of Xenocrates, a successor of Plato as teacher in the Academy, to the effect that he declined to admit a pupil without mathematical training by saying, "Depart, for thou hast not the grip of philosophy."

According to Plutarch, Plato held that God was a great geometer, that for that reason Plato made a knowledge of geometry an indispensable prerequisite for the study of philosophy.

The statements of Tzetzes, Diogenes Laertius and Plutarch converge to the same central idea. Moreover, if we remember that quantities of Greek manuscripts have been lost, it seems quite probable that the alleged inscription is authentic. If it is apocryphal and must be placed with the story of William Tell or the premature fall of the Washington cherry tree, it nevertheless conveys a thought which is in complete harmony with Plato's teaching.

Aristotle has left no direct, explicit dictum on the educational value of arithmetic and geometry, even though his writings teem with reasoning mathematical in character. Perhaps the nearest approach is his statement that mathematics makes manifest *order* and *symmetry*. Aristotle cultivated mathematics without expressing himself on the disciplinary value any more than did the great professional Greek mathematicians, Euclid, Archimides, Apollonius of Perga and Diophantus.

Socrates is reported by Xenophon, in his "Memorabilia" (Book IV., Ch. 7), as strongly favoring the study of mathematics in so far as it is needed and useful in practical life. Socrates did not touch the question whether mathematics does or does not strengthen the rational powers generally. His center of interest lay in the fields of ethics and religion.

There were two sects among the ancients—the Pyrrhonics and the Epicureans—that were enemies of mathematics. The Pyrrhonics were skeptics and opposed all knowledge. Epicurus rejected the mathematical sciences because his opinions regarding nature were opposed to what was taught by mathematics.

It is worthy of notice that neither sect attacked mathematics on the ground that it did not train the mind. Apparently this question did not come before them for consideration.

Looking, therefore, upon Greek antiquity as a whole, we see that philosophers of great eminence, like Plato and some of his disciples, placed extraordinary emphasis upon the mind-training value of mathematics as a preparation for dialectical studies, and that *none* of the Greeks denied that value.

UNIVERSITY OF CALIFORNIA.
BERKELEY, CALIF.

NEW BOOKS.

Elementary Algebra. By J. L. NEWFELD. Philadelphia: P. Blakiston's Sons & Co. Pp. 383.

This book is designed to be both elementary and comprehensive. Factoring is particularly stressed and a table of logarithms is added.

Lippincott's Classics. Edited by EDWIN L. MILLER.

Coleridge's The Rime of the Ancient Mariner and Other Poems. Notes by LOUISE POUND. Pp. 144.

Burke's Speech on Conciliation with America. Notes by EDWIN L. MILLER. Pp. 120.

Tennyson's Idylls of the King. Notes by WILLIS H. WILCOX. Pp. 183.

Shakespeare's A Midsummer-Night's Dream. Notes by CLARENCE STRATTON. Pp. 116.

A Second Book in Algebra. By FLETCHER DURELL AND E. E. ARNOLD. New York: Charles E. Merrill Co. Pp. 330.

Space and Time in Contemporary Physics: An Introduction to the Theory of Relativity and Gravitation. By MORITZ SCHLICK. New York: Oxford University Press. Pp. vi + 87. $2.50.

The author writes as a metaphysician who gladly embraces the theories of Einstein as affording a more acceptable interpretation of the universe than was previously possible. He reviews special and general relativity, relativity of space, relativity of motions, the fundamental postulate of relativity, and the fundamental law of the new theory which is, "the world-line of a material point is a geodetic line in the space-time continuum," and concludes with the finitude of the universe and relations to philosophy.

According to this theory space and time are nothing in themselves. They are the mere shadows of passing objects and events. They have no existence except in connection with objects and events, are not absolute but relative. If all dimensions of objects were changed in the same ratio, we should not know it, asserts this theory in preparing the way for the acceptance of its own views. A point transformation of space with a unique inverse and admitting of certain invariants as in measurements of dimensions would present two spaces such that we would not be able to know the difference between them. Thus we would be able to affirm at pleasure the one or the other for the interpretation of our experiences. Thus the way is clear for Einstein. As time and space are connected together in every external experience every object has a four-dimensional existence, a world-line in the time-space continuum.

In connection with the finitude of the world, a distinction is drawn

between "endless" and "infinite." Thus space is endless but not infinite, which gives satisfaction or not according to the tastes and inclinations of the particular philosopher.

The following quotation gives a good idea of the author's position. "The structure of the universe, which the general theory of relativity unveils to us, is astounding in its logical consistency, imposing in its grandeur, and equally satisfying for the physicist as for the philosopher. All the difficulties which arose from Newton's theory are overcome; yet all the advantages which the modern picture of the world presents, and which elevate it above the view of the ancients, shine with a clearer luster than before. The world is not confined by any boundaries, and is yet harmoniously complete in itself. It is saved from the danger of becoming desolate, for no energy or matter can wander off to infinity, because space is *not* infinite. The infinite space of the cosmos has certainly had to be rejected; but this does not signify such sacrifice as to reduce the sublimity of the picture of the world. For that which causes the idea of the infinite to inspire sublime feelings is beyond doubt the idea of the endlessness of space (actual infinity could not in any case be imagined); and this absence of any barrier, which excited Giordano Bruno to such ecstasy, is not infringed in any way.

"By a combination of physical, mathematical, and philosophic thought genius has made it possible to answer, by means of exact methods, questions concerning the universe which seemed doomed for ever to remain the objects of vague speculation. Once again we recognize the power of the theory of relativity in emancipating human thought, which it endows with a freedom and a sense of power such as has been scarcely attained through any other feat of science."

With this panegyric some will agree, but there will be many who, because of the relativity and endlessness (not to say infinity) of speculation, will be loathe to believe that we have here reached the finitude of theorizing, or anything like a final theory of the universe.

Some Famous Problems of the Theory of Numbers. By G. H. HARDY. Oxford: The Clarenden Press. Pp. 34.

Laird & Lee's Vest Pocket Littre-Webster Dictionary. English—French, French—English. Contains 60,000 words, meanings and idioms. 290 pp. Leather. Price $1.00.

Laird & Lee's Vest Pocket Standard Dictionary. English—Spanish, Spanish—English. 375 pp. Officially endorsed by the United States government. Contains maps of the Spanish-speaking countries. Leather. Price $1.00.

NOTES AND NEWS.

Meeting of Southern Section.

Members of the Association of Teachers of Mathematics in the Middle States and Maryland should observe that the $1.00 (or more if in arrears) they send now pays their dues in the association to December, 1921 and their subscription to The Mathematics Teacher for four issues (September and December, 1920, and January and February, 1921). After the February issue their subscription to the magazine should be paid to the National Council of Teachers of Mathematics (Mr. J. A. Foberg, 3829 N. Tripp Ave., Chicago, Ill., being the treasurer), at $2.00 a year of eight issues. All members of our association are urged to continue their subscription as they will find that the magazine will be of greater helpfulness than ever.

The spring meeting of the "Southern Section" was held in Washington May 1. The morning session consisted of demonstration lessons in algebra and geometry by Wiliam Betz, Lincoln School, Columbia University, New York City; followed by a lecture on "Motivation of Introductory Lessons." The demonstration lessons were given to classes from the high schools of Washington, and they created unusual interest and inspiration.

The afternoon program was given up to the consideration of the "Junior High School Movement." William A. Wetzel, Principal of the High School, Trenton, New Jersey, gave a very able and helpful address on "The Junior High School Movement with Reference to the Teaching of Mathematics."

The officers of the Section are: *Chairman,* Harry English; *Secretary and Treasurer,* Ida Hammond.

National Committee on Mathematical Requirements.

The National Committee on Mathematical Requirements held a meeting at Lake Delavan, Wisconsin, on September 2, 3 and 4,

at which a number of reports were discussed and adopted. A report on The Revision of College Entrance Requirements received the greatest amount of discussion. It is hoped that this report may be released for publication early in October. It includes a general discussion of the present problems connected with college entrance requirements in mathematics, a report of an investigation recently made by the National Committee concerning the value of the various topics in elemenatry algebra as preparation for the elementary college courses in other subjects and a suggested revision of the definitions of entrance units in elementary algebra and plane geometry. In connection with the suggested requirements in plane geometry, a list of fundamental propositions and constructions is attached. This list includes the proposition which may be assumed without proof or given informal treatment, a list of the most fundamental theorems and constructions from which it is intended that questions on entrance examination papers other than originals be chosen and a list of subsidiary theorems. It is proposed to prepare a mimeographed edition of this list of propositions and constructions at the earliest possible moment for the benefit of such teachers as may desire to make use of it in connection with their classes during the coming year. A copy will be sent to any person interested upon application to the Chairman of the Committee (J. W. Young, Hanover, New Hampshire).

A preliminary draft on Mathematics in Experimental Schools was discussed at this meeting. Mr. Raleigh Schorling, of the Committee, has spent over a year collecting material for this report. It is hoped that it will be ready for publication early next spring. The report will be an extensive one and will describe in detail the work actually done in mathematics in experimental schools throughout the country.

Miss Vevia Blair, of the Committee, presented her report on the Present Status of Disciplinary Values in Education. It is expected that this report also will be released for publication early in October. It gives a critical review of the complete literature concerning the experimental work on the transfer of training as well as an evaluation of this literature terminating in the formulation of certain propositions concerning disci-

plinary values which appear justified by the experimental work. A particularly valuable feature of the report would seem to lie in the fact that a large majority of the most prominent psychologists in the country appear to be ready to subscribe to the propositions formulated.

Professor E. R. Hedrick presented a report which he prepared at the request of the National Committee on "The Function Concept in Secondary School Mathematics." This report also will be published in the near future and is intended ultimately to form a part of the final report of the Committee on the Reorganization of the First Courses in Secondary School Mathematics. (A preliminary report on this subject was published for the Committee by the U. S. Bureau of Education last February as Secondary School Circular No. 5.)

A preliminary report on Junior High School Mathematics is in the press of the U. S. Bureau of Education and should be ready for distribution early in October. The National Committee desires the assistance of its cooperating organizations, which now number about 70, in the revision of this preliminary report. Comments, suggestions and criticisms should be sent to the Chairman of the Committee not later than January 1st in view of the fact that the Committee expects to take up the formulation of its final report on this subject immediately after this date.

A subcommittee under the chairmanship of Professor C. N. Moore is preparing a report on "Eective Courses in Mathematics in Secondary Schools." A committee under the chairmanship of Professor David Eugene Smith is preparing a report on "The Standardization of Terminology and Symbolism" and Professor R. C. Archibald is preparing one on "The Training of Teachers." It is expected that all three of these reports will be presented for the consideration of the National Committee in October.

The work of the National Committee and its recommendations were discussed in teachers' classes at the summer sessions of colleges, universities and normal schoool throughout the country. Addresses on the work of the Committee were given as follows: by Mr. Raleigh Schorling at Harvard University, by Professor

E. R. Hedrick at the Universities of Texas and Oklahoma, and by Mr. J. A. Foberg at the Universities of Iowa and Minnesota.

Present indications point to the fact that the work of the National Committee will have a prominent place on the programs of most teachers' organizations throughout the country during the coming year. The National Committee stands ready as before to help in every possible way in the preparation of such programs and will be glad to furnish material for discussion. It will also be pleased to furnish speakers for such meetings to the extent of its ability.